and an explanation found for the results just given. In the meantime, the present paper must remain as a statement of facts.

In conclusion, we wish to express our best thanks to Sir J. J. Thomson for the great interest he has taken in this work, which was carried out in the Cavendish Laboratory, and also to Sir William Ramsay for the loan of the krypton and xenon, which alone made the research possible.

A New Method of Determining the Radiation Constant.

By G. A. Shakespear, M.A., B.Sc.

(Communicated by Prof. J. H. Poynting, F.R.S. Received November 21, 1911,—Read January 8, 1912.)

According to Stefan's law the rate of radiation of energy from a full radiator in surroundings at a temperature of absolute zero is $\sigma\theta^4$ ergs per cm.^{2} per sec., where θ is the absolute temperature of the radiator. If the radiator be in surroundings which are themselves full radiators, but at absolute temperature θ_1, the rate of loss of energy by radiation is taken to be $\sigma(\theta^4-\theta_1^4)$.

The classical determination of the constant σ is due to Kurlbaum,[*] who used a surface bolometer with a platinum-black surface. The rise of temperature of the bolometer when exposed to the radiation from an approximately full radiator or "black body" was observed. The radiation was then cut off, and an equal rise of temperature was produced by increasing the main current in the bolometer. It was assumed that the energy received per second from the radiator in the first case was equal to the energy received per second from the increase of current in the second case. The resulting value of σ was $5{\cdot}33\times10^{-5}$ ergs per cm.^{2} per sec. per deg.^{4}, or $5{\cdot}33\times10^{-12}$ watts per cm.^{2} per deg.^{4}.

Kurlbaum gives a summary of previous measurements of σ, and there appears little doubt that his method was a great advance on those of earlier workers, and that it was carried out with such a high degree of experimental skill as to justify confidence in the result.

Later work, however, has suggested that Kurlbaum's value is too low. Fery[†] in particular, using an entirely different method, in which he employed a receiver which was more nearly a full absorber than a plane surface of platinum black, got a much higher figure ($6 \cdot 3 \times 10^{-12}$ watts per cm.^{2} per deg.^{4}), and Todd,* from experiments on the conductivity of gases, deduced a value between those of Kurlbaum and Fery, viz., $5 \cdot 48 \times 10^{-12}$ watts/cm.^{2} deg.^{4}. Still more recently Fery and Drecq† have obtained the value $6 \cdot 51 \times 10^{-12}$ watts/cm.^{2} deg.^{4}.

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[*] 'Ann. d. Phys.,' 1898, vol. 65, p. 746.

[†] 'Compt. Rend.,' April 5, 1909.

The method to be described in the present paper suggested itself to the writer in an investigation into the relative radiation and conduction losses from a heated surface at the temperature of boiling water in surroundings at ordinary room temperatures. The method is not free from objection, but in the state of uncertainty of our knowledge of the constant it appears advisable to have a determination based upon experiments differing in their nature from those previously utilised.

Principle of the Method.—It is well known that a heated body placed in air in surroundings at a lower temperature loses heat in three ways: (*a*) by conduction; (*b*) by convection; and (*c*) by radiation.

If the rate of loss of heat by a body be observed in two cases, the only difference being that the nature of the radiating surface varies, other conditions remaining the same, we shall probably be nearly correct in assuming that losses (*a*) and (*b*) will be the same, and that the difference between the observed rates of loss of energy in the two cases will be due to the difference of radiation losses only. If now these two different surfaces, at the temperature of boiling water, be exposed in turn to a radiomicrometer at the room temperature, we get the ratio of the rates of reception of energy from the two surfaces by the vane of the radiomicrometer.

Let us suppose the hot body to be a plane metal plate at 100° C. embedded in some badly conducting material, the upper surface of the plate being left free.

First let the free surface be lamp-blacked and maintained, by electrical or other means, at a temperature of 100° C. ($= \theta_1$ absolute). If we expose this surface to the radiomicrometer, the receiving surface of which consists of a lamp-blacked copper vane at a temperature of 15° C. ($= \theta_2$ absolute), we shall get a certain deflection $= D_1$.

Let $R_{B_1B_2}$ be the energy radiated per second from a square centimetre of the blackened hot-plate at temperature θ_1 when exposed to the blackened cold surface of the vane at temperature θ_2 (these conditions are indicated by the suffixes), and let $R_{B_2B_1}$ be the energy radiated per second from a square centimetre of the blackened vane at temperature θ_2 to the hot blackened plate at θ_1.

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* 'Roy. Soc. Proc.,' 1909, A, vol. 83.

† Fery and Drecq, 'Journ. de Phys.,' July, 1911, p. 558.

Then $D_1 = M(R_{B_1B_2} - R_{B_2B_1})$, where M is a constant. Now let the lamp-black be removed from the hot-plate, and let the surface of the plate be polished, raised to the temperature θ_1, and exposed, as before, to the radio-micrometer. Let the deflection be D_2. Then if $R_{P_1B_2}$ be the energy radiated per second from a square centimetre of the polished surface at θ_1 to the blackened surface at θ_2, we have

$$D_2 = M(R_{P_1B_2} - R_{B_2P_1}). \quad \text{Hence} \quad \frac{D_1}{D_2} = \frac{R_{B_1B_2} - R_{B_2B_1}}{R_{P_1B_2} - R_{B_2P_1}}.$$

Let $\frac{D_1}{D_2} = K$, hence $\frac{K-1}{K} = \frac{(R_{B_1B_2} - R_{B_2B_1}) - (R_{P_1B_2} - R_{B_2P_1})}{R_{B_1B_2} - R_{B_2B_1}}$.

Now let the hot-plate be again blackened and placed beneath, and at a short distance from, an extended parallel surface of lamp-blacked copper at temperature θ_2, the hot-plate being kept at temperature θ_1 by electrical or other means. Then if C represent the combined conduction and convection losses per second, and $L_{B_1B_2}$ the total loss of energy per second by the hot-plate in these conditions (*i.e.* the energy supplied by the electric current), we have

$$L_{B_1B_2} = C + A(R_{B_1B_2} - R_{B_2B_1})(1-b),$$

where A is the area of the radiating surface of the hot-plate and b is the reflection coefficient of the cold surface.

The factor $(1-b)$ is necessary, because the hot-plate will receive back a small fraction b of the radiation which it sends to the cold surface.

If the blackened surface be now replaced by the polished one, we have

$$L_{P_1B_2} = C + A(R_{P_1B_2} - R_{B_2P_1})(1-b) \text{ nearly.}$$

(This is not quite true, but a more detailed investigation, to be given later, shows that, in the circumstances, it is sufficiently nearly correct.)

Let $L_{B_1B_2} - L_{P_1B_2} = E$. Then

$$\begin{aligned} E &= (1-b)A\{(R_{B_1B_2} - R_{B_2B_1}) - (R_{P_1B_2} - R_{B_2P_1})\} \\ &= (1-b)A . \frac{K-1}{K} . (R_{B_1B_2} - R_{B_2B_1}) \\ &= (1-b)A . \frac{K-1}{K} . \sigma'(\theta_1^4 - \theta_2^4), \end{aligned}$$

so that

$$\sigma' = \frac{K}{K-1} \cdot \frac{E}{(1-b)A(\theta_1^4 - \theta_2^4)}.$$

We have assumed that $R_{B_1B_2} - R_{B_2B_1} = \sigma'(\theta_1^4 - \theta_2^4)$. It is necessary, therefore, to obtain σ from σ', and to do this we may expose in turn to the radiomicrometer a full radiator (*i.e.* a small hole in a constant temperature enclosure) and the lamp-blacked surface, used both at the same temperature.

Probably we may assume that the ratio of the deflections gives the ratio σ/σ'.

Experimental Details.—The quantities to be determined experimentally are thus E, K, *b*, and σ'/σ. Of these, all except E can be obtained from experiments with the radiomicrometer. We will consider first the determination of E.

We have to measure the rate of loss of heat from the blackened and the polished surfaces respectively when these are kept at a known temperature of about 100° C. in surroundings at about 15° C. This might be done in many ways, but the following three methods suggested themselves as likely to prove suitable: (*a*) by observing the rate of cooling of a cylindrical or spherical vessel containing a liquid at temperatures in the neighbourhood of 100° C.; (*b*) by passing steam into a spherical vessel and weighing the steam condensed in a given time; and (*c*) by keeping a surface hot by electrical means and determining the loss of energy by applying Joule's equation. Of these methods (*a*) was found not to lend itself to accurate measurement; (*b*) had many advantages, and could probably have been made to give excellent results; but (*c*), which was found to be very convenient in working, was ultimately adopted.

In applying the electrical method of heating, considerable difficulty was found in keeping the whole of any large surface at a sufficiently uniform temperature. For this reason it was considered advisable to use a comparatively small area of about 100 sq. cm. The form of apparatus ultimately adopted, which for convenience we may designate the "hot-plate," was constructed in the following manner:—

Two square plates, each of 10 cm. edge, were cut from a sheet of copper 0·3 cm. thick. These were heavily plated with silver, one side of each being polished. Two square sheets of mica, each of about 10·2 cm. edge, were also cut. The mica sheets were laid on one of the copper plates, and a row of holes was drilled through the copper and the mica along each of two opposite sides of the square, very near to the edges, the holes being equally spaced. One of the mica sheets was now removed, and short pegs of porcelain were fitted into the holes in the copper plate, forming a number of equidistant teeth, which projected about 0·1 cm. above the surface of the mica which lay on the copper plate. Bare manganin wire was wound backwards and forwards across the plate round these pegs, so as to form a sort of grid of equidistant parallel bars stretching across the plate about 0·25 cm. apart.

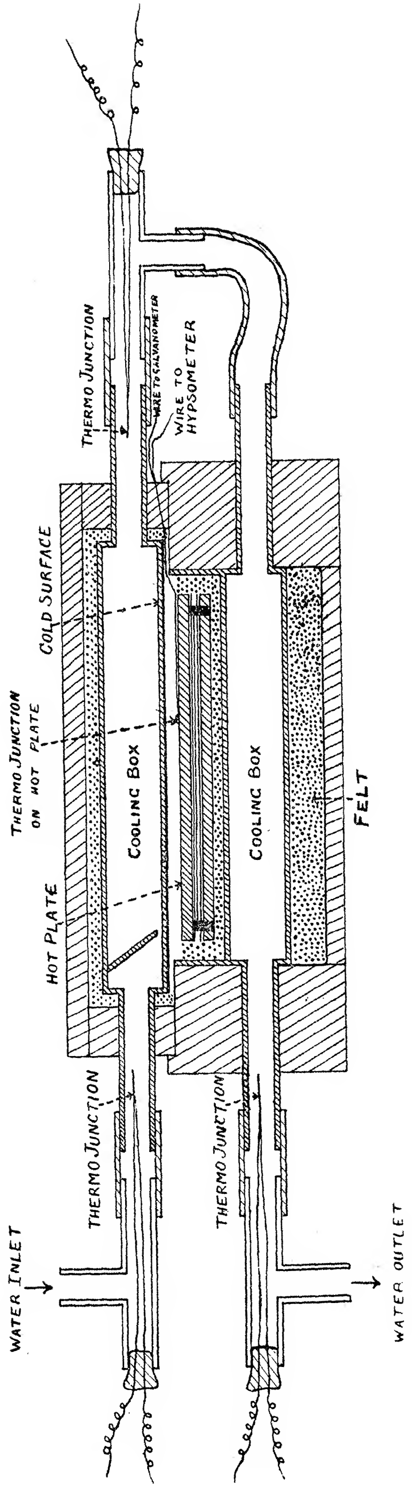

Fig. 1.

The other sheet of mica was now placed over the grid, the holes in the mica fitting on to the upper ends of the pegs. Over the mica the upper copper plate was laid, and was screwed tightly to the lower plate with two small screws. The whole formed a compound plate about 0·7 cm. thick, of which the outer surfaces were silvered and highly polished. The manganin wire in the middle was insulated from the copper by means of the mica. This grid constituted the electric heater.

A square flat box of copper was made, about 12 × 12 × 1·5 cm., the sides of which projected upwards above the top about 1·5 cm. all round, forming a sort of dish, in which the plate was laid on felt, so that the upper surface of the plate was parallel to and slightly below the tops of the edges of the box. The edges of the plate were also packed round with the felt up to the level of the top of the plate. The hollow box itself (forming the bottom of this dish) had an inlet and an outlet, through which water could be made to circulate. A baffle-plate was arranged inside this box to ensure that the water swept well along the upper surface of the cavity. This copper box containing the plate was fitted into a wooden frame, to which were attached two terminals, and to these terminals the ends of the wire of the heater were soldered.

The cold surface formed the lower face of another flat copper box about 15 by 15 cm., through which water circulated before passing to the lower box. This cooling box was surrounded with felt and fitted into a wooden frame. When this frame was laid on that of the hot-plate, the hot and cold surfaces were parallel and at a distance of about 0·4 cm. apart. Fig. 1 shows the arrangement in section.

Temperature Measurements.—The temperature of the exposed surface of the hot-plate was kept at the boiling point of water by means of the electric heater. To enable this temperature to be kept accurately, a thermocouple of thin copper and constantan wires was used, one junction being kept in the steam of boiling water in a hypsometer, while the other was kept in contact with the surface of the hot-plate. At first this second junction was soldered to the plate, but it was found more convenient later to screw it down to the surface by means of a small screw. The point of contact was about half-way between the centre of the plate and the middle of one edge. The couple was used in connection with a moving-coil galvanometer, which gave about 800 scale divisions for one degree difference of temperature of the junctions. Subsidiary experiments showed that the temperature of the plate could be kept steady at the boiling point with a maximum difference of temperature between different parts of the plate of less than 0·05° C. By the application of an adjustable safety valve the temperature of the steam could be kept at 100° C. when the barometer was below 76 cm., and a sort of negatively acting safety valve in connection with a water pump enabled the temperature to be kept at 100° C. when the barometer was above 76 cm. It was found best in practice, however, to have the water boiling under atmospheric pressure and to calculate the boiling point from the height of the barometer.

It was necessary to ascertain whether, when the galvanometer indicated thermal equilibrium between the junction in the steam and that on the plate, the plate was really at the temperature of the steam. This is a very difficult problem, and I do not know of any means whereby the temperature of a hot radiating surface can be determined with certainty. A solution was sought in the following manner: A thermocouple of thin constantan and copper wires was made, the two junctions being as similar as possible. Each of these junctions was laid on the centre of the flat end of a cork, the constantan wire being bent down along one side of the cork and the copper wire along the other side, and the two wires were then bound to the cork with string. The two corks were as similar as possible.

To obtain a surface the temperature of which might be presumed to be extremely near the boiling point, a hollow copper box in the form of a cylinder about 8 cm. diameter and 3 cm. deep, was heated by passing through it a current of steam. The hot-plate was then heated electrically until the thermo-junction attached to it was in equilibrium with the junction in the hypsometer, as indicated by the above-mentioned galvanometer. The two corks, with their thermo-junctions attached, were now taken and pressed one on the steam-heated surface and the other on the hot-plate, the difference of temperature of the two junctions being observed by the deflection of a galvanometer to which they were connected. This galvanometer gave a deflection of 500 divisions of the scale for 1 degree difference of temperature of the junctions. The two junctions were then interchanged, the one which was previously on the hot-plate being now placed on the steam-heated surface, while the one from the steam-heated surface was transferred to the hot-plate. This was done to compensate for any want of symmetry between the two junctions. In this way it was found that the temperature of the hot-plate was about 0·20° C. higher than that of the steam-heated surface. Hence a correction has to be made by adding 0·20° to the temperature of the steam to obtain the temperature of the hot-plate when its thermo-junction is in equilibrium with that in the hypsometer. One would expect some such effect on account of the conduction of heat along the wires from the junction.

The temperature of the cold surface was determined by means of a thermometer placed in the outflowing stream of water. The temperature indicated by this thermometer was slightly higher than that of the water in contact with the cold plate, because the stream after leaving the cold plate had circulated under the containing box of the hot-plate, absorbing during this circulation the heat conducted through the felt from the hot-plate. Thermo-junctions placed in the stream (*a*) at the entrance to the cold plate, (*b*) at the exit from the cold plate, and (*c*) near the thermometer at the exit from the box containing the hot-plate, enabled one to calculate the difference of temperature of the stream in the middle of the cold plate and at the thermometer. It was found in this way that the mean temperature of the water in its passage over the cold plate was 0·15° lower than that of the final efflux.

The rate of flow of the water was kept as nearly as possible constant throughout the experiments.

It was next necessary to ascertain whether the mean temperature of the water passing over the cold plate accurately represented the mean temperature of the cold plate itself. In a somewhat similar case Todd* found a difference of 0·4° between the temperature of the water and that of the plate, and he found that this difference was independent of the rate of flow. In the present experiment a difference of 0·29° C. was found, the plate being 0·29° C. warmer than the water flowing over it. The difference depended upon the rate of flow of the water, and for less rapid rates of flow than that employed the difference exceeded 0·5° C. (The discrepancy between this result and that of Todd can be partly accounted for, I think, by the form of his apparatus, in which the tube carrying the outflowing water passed through the inflowing stream.) The temperature of the cooling stream could be raised in cold weather by heating, with gas, a copper tube through which the water passed on its way to the cold plate. In later experiments, however, the temperature of the water in the mains rose to over 20° C., and it was found most convenient to use the water at the temperature at which it came, and if the temperature in the experiment with the black hot-plate differed from that with the polished one a correction was made, the amount of the correction having been determined from separate experiments. If this variation of temperature was considerable the experiments were rejected.

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* 'Roy. Soc. Proc.,' 1909, A, vol. 83.

The thermometer used in measuring the temperature of the efflux water was compared with an air thermometer and found to be 0·05° C. too low in the neighbourhood of the temperatures used.

The Radiating Surfaces.—The object to be attained was the provision of a surface which could be altered with ease from one of high radiating properties to one of low.

In the calculation the fraction $K/(K-1)$ occurs, where K is the ratio of the deflections of the radiomicrometer when the two surfaces are successively exposed to it. It is evident that if K be great compared with unity, this fraction will become sensibly equal to 1, and great accuracy in determining K is not required. Hence it was found convenient to use a highly polished surface of silver, which, when required, could be converted into a black surface by holding it in the smoke of a flame of benzene. The radiating copper plate was therefore thickly plated with silver, which was polished as highly as possible. When it was to be smoked the plate was held in position by two temporary clamps, one at each of two opposite corners. A good deal of practice was necessary to enable one to give the surface a uniform coat of soot of about the same thickness each time. The surface of the cold plate also was blackened with benzene smoke. Both the cold surface and the radiomicrometer vane were of copper, blackened in this way.

Measurement of the Energy Supplied.—The current in the heating grid was measured with a Weston ammeter of excellent construction, recently readjusted by Messrs. Elliott, and capable of being read to 0·001 ampère. The instrument was tested with a potentiometer for the range of current used and no error could be detected; it is probable therefore that the current measurements are correct to 0·001 ampère. The resistance of the grid was measured with sufficient accuracy with a resistance box made by the Cambridge Scientific Instrument Company.

Determination of K, *b, and* σ'/σ.—For the purpose of these comparisons a Boys radiomicrometer was used, the receiving vane being a plane surface of thin copper blackened with soot. This, of course, did not constitute a full absorber, and it was therefore thought advisable to check the results obtained therefrom with those yielded by a different apparatus, in which a more complete approximation to a full receiver was attained. This consisted of a single thermo-couple of Sb-Bi (each strip being about 0·3 × 0·05 × 0·01 cm.) one junction of which was soldered to a thin copper disc 0·2 cm. diameter. The disc was blackened and placed in a slightly larger hole in the centre of a brightly silvered disc of about 2 cm. diameter, which formed the diametric plane of a hemispherical cavity 2 cm. in diameter. The disc and hemisphere were of copper, silvered and highly polished. A hole 0·2 cm. in diameter, at the pole of the hemisphere, admitted radiation from the hot body to the receiving disc of the thermo-couple. Thus, any radiation falling on the black disc and diffusely reflected would be, to a large extent, reflected back to the disc. This hemisphere was mounted inside a brass tube fitted with a series

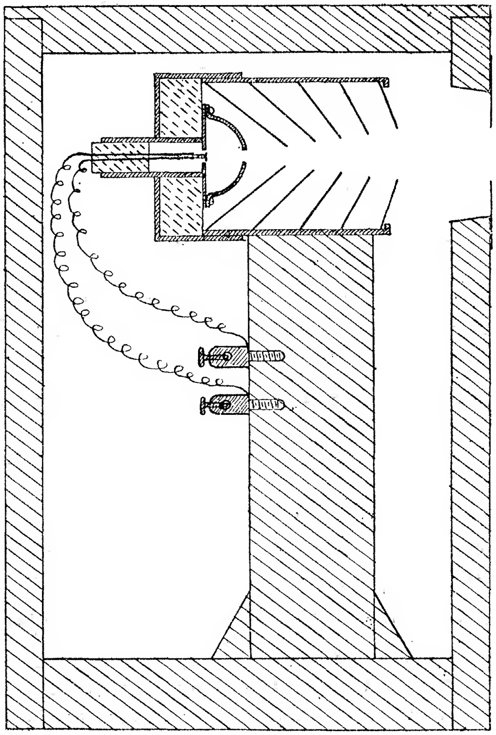

Fig. 2.

of conical diaphragms of silvered copper blackened on the side facing the hemisphere, and brightly polished on the side facing the hot body. The arrangement is shown in section in fig. 2. The whole was placed inside a protecting wooden box and used in conjunction with a low resistance Broca galvanometer. A comparison of the radiation from the hot surfaces with this instrument gave results sensibly the same as those given with the radiomicrometer; the latter, however, being much more convenient in practice, was afterwards always used for measuring the quantity K. The agreement between the two methods of comparison would seem to show that the selectiveness of the absorption exercised by the lamp-blacked copper vane of the radiomicrometer is very slight for the wave-lengths used.

A large water-cooled screen, with an aperture 2 cm. in diameter, protected the radiomicrometer from the heat of the hot-plate. The aperture could be opened or closed by means of a water-cooled shutter worked from a distance with a lever.

The ratio K was found to be usually between 51 and 57, the latter figure being given when the silver surface was most highly polished and the lamp-black coat at the most effective thickness.

Relation between the Lamp-black Surfaces and the Full Radiator.—For the purpose of finding the best thickness of soot, a steam-heated flat box of copper, one face of which could be smoked, was used. The results obtained agreed fairly well with those of Kurlbaum,* the maximum effect being given with a layer of about 40 mgrm./dm.^{2}. The deflection of the radiomicrometer due to the blackened surfaces was compared with that produced by the radiation from a steam-heated "black body" or full radiator. This "black body" was made in the form of a cylinder, with hollow walls, in which steam circulated. The internal diameter was 31 cm. and the length was about the same. The ends of the cylinder were closed with two hollow discs, one of which had a central circular aperture of 2·3 cm. diameter. The object of constructing the "black body" in sections was to enable it to be smoked all over the interior surface. The hollow vessel thus built up was enclosed in a double thickness of thick felt, in which an opening was left over the aperture of the cavity. The curved part and the two end pieces had each a separate inlet and outlet tube for the steam. When the "black body" was used with the radiomicrometer, diaphragms were arranged so that the receiving surface received radiation from the interior of the cavity only. In consequence of the relative smallness of the aperture as compared with the whole of the internal surface, this was probably a close approximation to the theoretical full radiator.

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* 'Ann. d. Phys.,' 1899, vol. 67, p. 846.

An unexpected fact, namely, that *the radiation from a lamp-black surface depends upon the nature of the surface upon which the lamp-black is deposited*, was revealed in the course of these experiments in the following way :—

When the steam-heated blackened copper radiator had been given a thickness of soot which gave the maximum radiation, the silvered hot-plate was smoked and raised to the boiling point, as indicated by the thermo-junction, and then exposed to the radiomicrometer. It was not found possible to get so great an effect from this plate as from the steam-heated blackened copper surface. It was suspected that this might be due to some defect in the means of measuring the temperature of the surface of the plate, and that the plate might be below the boiling point. A flat copper box was therefore made, with steam inlet and outlet, the two opposite faces being as similar as possible. One of these flat faces was silvered and brightly polished, the other being left in the natural condition of dull unpolished copper. Both faces were then smoked, and the radiations from them at the boiling point were compared. It was found here also that the smoked silvered surface gave a less effect than the smoked copper, the former producing sensibly the same effect as the smoked silvered hot-plate. Perhaps the explanation is to be found in the inability of both lamp-black and polished silver to emit waves of certain lengths, the lamp-black being transparent to them and the silver perfectly reflecting.

The maximum effect of smoked copper at 100° C. was found to be nearly 98 per cent. of that of the full radiator, whereas the maximum effect of smoked polished silver was only about 95 per cent. Hence the reflection coefficient (b) of the blackened copper cold surface may be taken as about 0·025, and the maximum value of σ'/σ was 0·95. It may be observed that Kurlbaum* found that lamp-blacked platinum gave a maximum of about 95 per cent. of the radiation of a "black body," and he attributed this to the temperature gradient in the body of the lamp-black. In view of the present experiments, this conclusion would seem to require some revision.

Method of Carrying Out a Determination of σ.—We are now in a position to consider the course of a typical experiment for the determination of σ. The procedure was as follows: A current, larger than that which would be required ultimately, was put on to the heater, and the gas was lighted under the hypsometer, which contained one of the junctions of the thermo-couple, the other junction of which was attached to the hot-plate. In about 10 minutes the water in the hypsometer boiled, and it was generally found that the hot-plate had reached a temperature rather higher than the boiling point. The water was now turned on to the cooling boxes, and the electric current was adjusted until the spot of light on the galvanometer scale remained practically stationary at the zero point. When the current was thus properly adjusted, an increase of 0·001 ampère caused the spot of light to move slowly to the right, and a decrease of 0·001 ampère caused the spot to move slowly to the left. When the conditions were favourable, the spot could be got to remain almost motionless for hours, but usually a slight change of temperature of the cooling stream, or a rise or fall of the barometer, would necessitate small changes of the current to maintain the steady condition. When this steady state was reached, the temperature of the outflowing water was read, and the height of the barometer was observed.

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* 'Ann. d. Phys.,' 1899, vol. 67, p. 846.

The hot-plate was now exposed to the radiomicrometer, and the mean of about twenty deflections due to successive exposures of equal duration separated by equal intervals of time (20 seconds) was observed. If the plate had previously been bright the current was disconnected and the surface was smoked; the current was again turned on, the temperature of the plate was raised to the boiling point, and the plate was again exposed to the radiomicrometer as before. The cold surface was then fitted over the hot-plate, and the current was again adjusted until the steady state was reached.

The soot was then swept off the plate with a camel hair brush and weighed. The plate was again polished with a little very soft cotton wool, replaced under the cold plate, and the current once more adjusted to obtain the steady state. Immediately before or after the exposure of the plate to the radiomicrometer the " black body " was exposed to the instrument and the deflection observed.

It was found that the polished silver gave practically always the same deflection as compared with that given by the " black body," and so this part of the experiment was often omitted in the later work. As some time was required for the " black body " to reach the boiling point, it was found convenient to make the comparison of the plate with a standard lamp-black consisting of the flat steam-heated copper box above mentioned, smoked so as to give its maximum radiation. This standard itself was compared with the " black body " from time to time.

The area of the hot-plate was found by measurement with a travelling microscope.

In the latest experiments the plate was fixed in its box with a small peg of ebonite at each corner, the pegs also serving as spacing pieces to secure constancy of distance between the hot and cold surfaces. One advantage of this arrangement was that the hot plate could be used either under or over the cold plate; in the latter case there would be presumably no convection, whereas in the former some might occur. Experiments made with the hot plate first below and then above the cold plate showed that the effect of convection was extremely small, the heat loss being due almost entirely to radiation and conduction.

The following data are the results of a typical experiment:—

Resistance of heater	13·26 ohms.
Area of plate	101·7 cm.^{2}.
K	57
Reflection coefficient of cold surface = b	0·025
Deflection of radiomicrometer by blackened hot plate	46·7 divs.
Deflection by "black body."	50·0 divs.
Hence σ'/σ	0·934
Temperature of polished surface (corrected)	99·9° C.
Temperature of cold surface (corrected)	19·6° C.
Balancing current	1·13 amp.
Temperature of blackened surface (corrected)	99·9° C.
Temperature of cold surface (corrected)	19·6° C.
Balancing current	1·321 amp.
Thickness of layer of lamp-black	40 mgrm./dm.^{2}

Then

$$\sigma' = \frac{\mathrm{K}}{\mathrm{K}-1} \cdot \frac{\mathrm{E}}{\mathrm{A}\,(372{\cdot}9^4 - 292{\cdot}6^4)\,(1-b)}$$

$$= \frac{57}{56} \cdot \frac{(1{\cdot}321^2 - 1{\cdot}130^2)\,13{\cdot}26}{101{\cdot}7\,(1{\cdot}201 \times 10^{10}) \times 0{\cdot}975} = 5{\cdot}30 \times 10^{-12}\ \text{watts/cm.}^2\ \text{deg.}^4,$$

so that $$\sigma = \frac{5{\cdot}30 \times 10^{-12}}{0{\cdot}934} = 5{\cdot}67 \times 10^{-12}\ \text{watts/cm.}^2\ \text{deg.}^4.$$

The value of σ ultimately obtained is derived from six experiments in which the conditions were so favourable that no correction for change of temperature of the cooling stream or of the steam was necessary. The data for these experiments are given on p. 193.

Twenty experiments, which were rejected because the height of the barometer or the temperature of the cooling stream varied during the operation, gave results varying between the extreme values $5{\cdot}83 \times 10^{-12}$ and $5{\cdot}49 \times 10^{-12}$, with a mean of $5{\cdot}66 \times 10^{-12}$.

The present work may therefore be regarded as giving the value $\sigma = 5{\cdot}67 \times 10^{-12}$ watts per cm.^{2} per deg.^{4}, or $5{\cdot}67 \times 10^{-5}$ ergs per sq. cm. per sec. per deg.^{4}.

It may be of interest to observe that other experiments were made in which the polished plate was painted with aluminium paint, the two radiating surfaces being in this case the aluminium paint and the lamp-black. The value of K was very variable, but when the paint had been

Temp. of hot-plate (corr.).	Temp. of cold surface (corr.).	Balancing current.	Ratio σ'/σ.	Thickness of lamp-black.	σ'.	σ.
° C.	° C.	ampères.		mgrm./dm.^{2}	watts/cm.^{2} deg.^{4}	watts/cm.^{2} deg.^{4}
99·79	15·80	Black...... 1·310 Bright ... 1·115	0·920	74	$5{\cdot}21 \times 10^{-12}$	$5{\cdot}66 \times 10^{-12}$
99·94	16·00	Black...... 1·318 Bright ... 1·114	0·960	56	$5{\cdot}46 \times 10^{-12}$	$5{\cdot}67 \times 10^{-12}$
99·90	19·60	Black...... 1·321 Bright ... 1·130	0·934	40	$5{\cdot}30 \times 10^{-12}$	$5{\cdot}68 \times 10^{-12}$
100·24	20·20	Black...... 1·319 Bright ... 1·129	0·934	40	$5{\cdot}28 \times 10^{-12}$	$5{\cdot}65 \times 10^{-12}$
99·90	20·00	Black...... 1·313 Bright ... 1·122	0·936	44	$5{\cdot}29 \times 10^{-12}$	$5{\cdot}65 \times 10^{-12}$
99·90	19·45	Black...... 1·316 Bright ... 1·123	0·950	43	$5{\cdot}38 \times 10^{-12}$	$5{\cdot}66 \times 10^{-12}$
				Mean value of σ........		$5{\cdot}67 \times 10^{-12}$

carefully washed it was about 4·3. Evidently an error in K is here much more serious than in the case previously described. Moreover, the difference E was much less, and the possible error had a much greater relative value. Consequently the value of σ deduced from these experiments varies widely, *e.g.*, three experiments in favourable conditions gave 5·47, 5·27, and $6{\cdot}25 \times 10^{-12}$, the mean being $5{\cdot}66 \times 10^{-12}$. Though no importance is attached to these experiments they may be regarded as a rough check upon the foregoing.

It may be observed that the value $5{\cdot}67 \times 10^{-12}$ is much nearer to the values given by Kurlbaum ($5{\cdot}33 \times 10^{-12}$) and Todd ($5{\cdot}48 \times 10^{-12}$) than to that of Fery and Drecq ($6{\cdot}51 \times 10^{-12}$). At the same time it should be remarked that Kurlbaum and Todd used the same range of temperature as that employed in the present work, whereas Fery dealt with much higher temperatures. On the other hand, other workers using methods similar to that of Kurlbaum have obtained values not very different from Kurlbaum's over greater ranges of temperature.* In the work of Fery and Drecq, however, if one may judge from the diagram representing the arrangement of their apparatus, there seems to be reason to expect a serious error in consequence of reflection of radiation from the inside of the aperture in the cork screen. No mention is made of any precaution to avoid such possible error, which, it may be noted, would result in too high a value for σ.

The following (for which I am largely indebted to my colleague, Dr. Guy Barlow) is a more detailed investigation of the theory of the experiment.

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* *Cf.* Valentiner, 'Ann. d. Phys.,' 1910, vol. 31, p. 275.

Let X and Y (fig. 3) represent the polished hot-plate and the blackened cold surface respectively, their reflection coefficients being p and b.

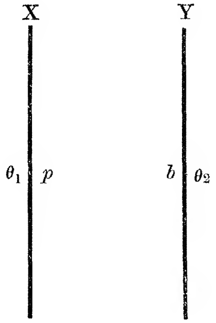

FIG 3.

Let a quantity of energy, Q, be radiated from X to Y. Of this quantity a fraction bQ will be reflected and $(1-b)$Q will be retained by Y. Of the reflected portion, pbQ will be again reflected and $(1-p)b$Q retained by X, and so on.

In the limit, X will thus receive back $(1-p)\,bQ/(1-bp)$ and Y will retain $Q\,(1-b)/(1-bp)$.

(It must be noted here that we are assuming that the coefficients b and p remain constant, which is probably not true. To make the investigation full we should require to know the value of each of these coefficients for each part of the spectrum of the radiation used. This being at present out of the question, we must make use of the approximation here adopted.)

Again, if a quantity of energy Q' be radiated from Y an amount $Q'(1-p)/(1-bp)$ will be absorbed by X, the factor $(1-p)/(1-bp)$ being the effective absorption coefficient of X.

Now, let us assume that each square centimetre of plate X at temperature θ_1 radiates $R_1\,(1-p)$, where R_1 = full radiation = $\sigma\theta_1{}^4$.

Then, as we have just seen, X will receive back altogether $\dfrac{b(1-p)}{1-bp}R_1(1-p)$. And if we assume that each square centimetre of Y radiates $R_2\,(1-b)$, where R_2 = full radiation for temperature θ_2, we shall have X receiving in addition $R_2\,(1-b)\,.\,(1-p)/(1-bp)$.

If, then, the temperature of X be kept at θ_1 by electrical means, and if C represent the rate of loss of energy by conduction and convection, the rate of supply of energy by the current must be

$$R_1\,(1-p)-R_1\,(1-p)\frac{b\,(1-p)}{1-bp}-R_2\,(1-b)\,\frac{1-p}{1-bp}+C.$$

Hence, in the notation previously used, if **X** be the polished hot plate, we have

$$\mathrm{L}_{\mathrm{P_1B_2}} = \mathrm{A}\left[\mathrm{R}_1(1-p)\left\{1-\frac{b(1-p)}{1-bp}\right\}-\mathrm{R}_2(1-b)\frac{1-p}{1-bp}\right]+\mathrm{C}$$

$$= \mathrm{A}\left[\frac{(1-p)(1-b)}{1-bp}(\mathrm{R}_1-\mathrm{R}_2)\right]+\mathrm{C}.$$

Similarly, if X be the blackened silver hot-plate, with reflection coefficient b', also at θ_1, we have

$$\mathrm{L}_{\mathrm{B_1B_2}} = \mathrm{A}\left[\frac{(1-b')(1-b)}{1-bb'}(\mathrm{R}_1-\mathrm{R}_2)\right]+\mathrm{C};$$

so that

$$\mathrm{E} = \mathrm{A}(1-b)(\mathrm{R}_1-\mathrm{R}_2)\left[\frac{1-b'}{1-bb'}-\frac{1-b}{1-bp}\right].$$

Now b is about 0·02 and b' about 0·05, while p is about 0·98. Hence we may neglect bb', so that

$$\mathrm{E} = \mathrm{A}(1-b)(\mathrm{R}_1-\mathrm{R}_2)\left(1-b'-\frac{1-p}{1-bp}\right)$$

$$= \mathrm{A}(\mathrm{R}_1-\mathrm{R}_2)\left(\frac{p-b'-2bp}{1-bp}\right)$$

$$= \mathrm{A}\sigma(\theta_1^4-\theta_2^4)\left(\frac{p-b'-2bp}{1-bp}\right).$$

The quantities p, b, and b' are given by the radiomicrometer experiments, which we may now consider.

Theory of Radiomicrometer Experiments.—We assume that the deflection of the instrument is proportional to the difference between the energy entering the radiomicrometer and that leaving it by radiation.

We also assume that the hot-plate at θ_1, when exposed to the radiomicrometer, is receiving from its surroundings full radiation at temperature θ_2.

This is probably nearly true, since the field of view, so to speak, of the hot-plate is nearly filled by the blackened surface of the large water-cooled screen in which is the small aperture admitting radiation to the radiomicrometer.

Let us now consider the quantities of energy Q_1, Q_2, Q_3, Q_4, and Q_5 respectively entering the instrument when we expose to it

(*a*) the polished silver hot-plate at θ_1,
(*b*) the blackened silver hot-plate at θ_1,
(*c*) the blackened copper surface at θ_1,
(*d*) the "black body" at θ_1,
(*e*) the "black body" at θ_2.

We have:—

(*a*) $Q_1 = M\{(1-p)R_1 + pR_2\}$,

(*b*) $Q_2 = M\{(1-b')R_1 + b'R_2\}$,

(*c*) $Q_3 = M\{(1-b)R_1 + bR_2\}$,

(*d*) $Q_4 = MR_1$,

(*e*) $Q_5 = MR_2$,

where M is a constant.

Now (*e*) gives zero deflection, therefore we may regard the energy emerging from the radiomicrometer as equal to that entering, $= MR_2$.

Hence if the deflections be respectively D_1, D_2, etc., we have

$$D_1 \propto Q_1 - Q_5 = G\{(1-p)R_1 - (1-p)R_2\} = G(1-p)(R_1 - R_2),$$

where G is a constant.

$$D_2 \propto Q_2 - Q_5 = G(1-b')(R_1 - R_2),$$

$$D_3 \propto Q_3 - Q_5 = G(1-b)(R_1 - R_2),$$

$$D_4 \propto Q_4 - Q_5 = G(R_1 - R_2).$$

Thus

$$\frac{D_1}{D_4} = (1-p) \quad \text{and} \quad p = 1 - \frac{D_1}{D_4},$$

$$\frac{D_2}{D_4} = (1-b') \quad \text{and} \quad b' = 1 - \frac{D_2}{D_4},$$

$$\frac{D_3}{D_4} = (1-b) \quad \text{and} \quad b = 1 - \frac{D_3}{D_4}.$$

The result obtained from the approximate theory given earlier is equivalent to

$$\sigma = \frac{E}{A(\theta_1^4 - \theta_2^4)(p-b')(1-b)}$$

$$= \frac{E}{A(\theta_1^4 - \theta_2^4)(p-b'-bp)},$$

whereas that here arrived at is

$$\sigma = \frac{E(1-bp)}{A(\theta_1^4 - \theta_2^4)(p-b'-2bp)}.$$

In consequence of the magnitudes of *p*, *b*, and *b'*, the two expressions are nearly identical, and the adoption of the formula given by the more detailed investigation would merely change the result from $5{\cdot}67 \times 10^{-12}$ to $5{\cdot}68 \times 10^{-12}$.

No mention has been made of any edge effect, because the "field of view" of every part of the hot radiating surface is practically filled with black surfaces at temperature θ_2.

Since the above was ready for publication my attention has been drawn to a paper by Bottomley and King,* of which I was not previously aware. The method used by these experimenters bears some resemblance to that described in the present paper, but the treatment and results differ widely.

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* 'Phil. Trans.,' 1908, vol. 208.